

Calculation and Reporting Measurement Uncertainties

What is a measurement uncertainty?

[Guide to the Expression of Uncertainty in Measurement \(GUM\)](#) defines the uncertainty of measurement as follows:

“parameter, associated with the result of measurement, that characterizes the dispersion of values that could reasonably be attributed to the measurand” (the artifact being measured or compared)

[International Vocabulary of Metrology \(VIM\)](#) defines the uncertainty of measurement as follows:

“parameter, characterizing the dispersion of the values being attributed to a measurand, based on the information used”

In practical terms, each time a measurement is taken the person making the measurement assigns a specific value to that specific artifact (measurand) at that specific time. If this measurement is repeated over time, there will be some variation in the assigned value. Uncertainty as a metrological concept is an attempt to quantify that measurement variation with the goal of ensuring a majority (approximately 95.45 % for our use cases) of these future measurements fall within the uncertainty range. The VIM also says measurement uncertainty can be interpreted as doubt about the true value of the measurand that remains after making a measurement.

Why does reporting measurement uncertainties matter?

No measurement method is perfect, and many factors can affect the artifact being measured. By reporting an uncertainty alongside the measured value, users gain clear insight into the expected range of variation associated with the measurement.

For example, the diameter of a US penny (now extinct) will vary based on the temperature of the penny itself. If a person measures a penny outside in the winter, then brings the coin inside to warm up before measuring it a second time, both measurements would be correct and accurate, but they will be different.

What is the difference between uncertainty and error?

Unlike uncertainty, which is a quantified estimate of measurement variation, error is an inaccuracy in the measurement which can be repeated.

What is the [Guide to Uncertainty Expression in Measurement \(GUM\)](#)?

The [GUM](#) is an internationally recognized publication which establishes general rules for creating, evaluating, and reporting measurement uncertainties. The [GUM](#) was prepared by a working group from the Bureau International des Poids et Mesures (BIPM), the International Organization for Standardization (ISO), the International Organization of Legal Metrology (OIML), among others to help standardize the quantification of uncertainty around the world.

How to build an uncertainty?

The process described below are the steps used to initially create a measurement uncertainty, but measurement uncertainty must be evaluated regularly (typically once per year) based on new data and information over time to ensure the calculated and reported measurement uncertainty remains valid.

1. Determine the components of an uncertainty

For most established SOPs, the components of the uncertainty budget are listed in the SOP itself. For laboratory developed procedures, the laboratory itself must consider factors which could contribute to the uncertainty of the measurements. Each component identified must be evaluated for its contribution to the final reported uncertainty.

Methods for evaluating these components are generally grouped into two types. Components relating to the measurement process itself are called Type A (statistically derived values from data) whereas components related to the standard(s) and other factors are labeled Type B (non-statistically derived values).

2. Quantify those components

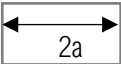
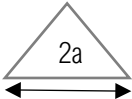
Uncertainties must be quantified to be useful in metrology. For example, when reading a prover meniscus, the reading can depend on the room lighting, the leveling of the artifact, the parallax caused by the magnifying glass, and many other factors. Each component of the uncertainty must be quantified in some way so it can mathematically contribute to the final *expanded uncertainty* (U) or omitted if determined to be insignificant.

3. Convert the components to a common unit of measure

Once all the components have been quantified the components need to be converted into a unit of measure which relates directly to the measurement made on the artifact (*estimated uncertainty* components). This is typically grams or milligrams for mass and gallons or cubic inches for volume. A good example is uncertainty in a temperature probe for volumetric calibrations: the uncertainty is in degrees Celsius, and this uncertainty must be transformed into a unit of volume.

4. Scaling to a common statistical distribution

These estimated uncertainty components must then be scaled based on the type of statistical distribution to *standard uncertainty* components with a sigma (σ) of 1. The scale factors for common statistical distributions are listed below.

Type of Statistical Distribution	Range	Scale Factor
Rectangular		$\frac{a}{\sqrt{3}}$
Triangular		$\frac{a}{\sqrt{6}}$
Reported Uncertainty		$\frac{U}{k \text{ factor}}$

5. Combining

Once all uncertainty components have been converted to the same units, they should be combined using the root sum squared (RSS) method into a *combined uncertainty* (u_c).

$$u_c = \sqrt{s_p^2 + u_s^2 + u_{c1}^2 + \dots}$$

6. Expanding & Rounding

The *combined uncertainty* must be multiplied by the coverage factor (k) to determine the *expanded uncertainty* (U). These values are unique to each laboratory.

$$U = u_c \times k$$

The coverage factor is based on the degrees of freedom (df) and the confidence interval (typically 95.45 %) for each measurement parameter. The degrees of freedom used to calculate the coverage factor is the number of measurements a laboratory had documented (N) on their control chart minus one for procedures involving a single measurement per artifact. Procedures with multiple measurements per artifact (such as NIST SOP 4) have larger degrees of freedom.

$$df = N - 1$$

This final *expanded uncertainty* is rounded to 2 significant figures in accordance with [NISTIR 6969](#) GLP 9 (2019).

7. Evaluating

Once a final rounded expanded uncertainty is developed, it should first and foremost make sense intuitively for the measurements – is it consistent with what you expect based on your measurement. Additionally, most documentary standards and procedures have criteria for mathematically evaluating the uncertainty based on any conformity assessment being made of the artifacts.

8. Reporting

The final expanded uncertainty and coverage factor must be reported on all calibration certificates issued along with a statement in general terms of how the uncertainty was calculated.

Practical Example

Let's consider the calibration of a hydrometer (a device used to measure the density of liquids) using a relatively simple method of comparing a known calibrated hydrometer with an unknown uncalibrated hydrometer by floating them both in a liquid (similar to water) one after another as a practical example of how to create and quantify uncertainty. This theoretical procedure requires the environment and liquid to be in the range of 20 °C to 23 °C and 40 % RH to 60 % RH and has 83 measurements documented in a control chart.

1. Determining the components for hydrometer calibration

Type A Components: Standard deviation of the process

Type B Components: Uncertainty of the calibrated hydrometer, and change in density of the liquid with temperature

2. Quantifying each component

Description	Abbreviation	Quantity	Source
Standard Deviation of the Process	s_p	0.023786443 °Baume (°Be)	This value is derived from a control chart based on replicate measurements
Uncertainty of the Standard	u_s	0.018 °Baume (°Be)	This value is based on the uncertainty received from the calibration laboratory. Typically, it is their expanded uncertainty divided by their coverage factor.
Liquid Density Variation	u_l	0.665597227288317 kg/m ³ Density of water at the range of temperatures. 998.213478 kg/m ³ at 20 °C 997.54788 kg/m ³ at 23 °C	This value is calculated based on the change in water density between the temperatures 20 °C to 23 °C because in theory the liquid could change temperature and therefore density during the procedure. Keep in mind this is only an assumption, and labs can and do make different assumptions about uncertainty components.

3. Converting each component to the measurement units

Both the standard deviation of the process and the uncertainty of the calibrated hydrometer are already in the units of the hydrometers (°Baume). Therefore, the only component that requires conversion is the liquid density variation which can be assumed to have a rectangular distribution.

To convert the liquid density variation from kg/m³ to °Baume the density of water at 20 °C and 23 °C must first be converted to °Baume.

$$^{\circ}Be = 145 - \left[\frac{145000}{\text{Density in kg/m}^3} \right]$$

$$^{\circ}Be (20^{\circ}C) = 145 - \left[\frac{145000}{998.213478} \right] = -0.259509373174239 ^{\circ}Be$$

$$^{\circ}Be (23^{\circ}C) = 145 - \left[\frac{145000}{997.54788} \right] = -0.356431364170646 ^{\circ}Be$$

Next take the difference in those two densities.

$$u_l = ^{\circ}Be (20^{\circ}C) - ^{\circ}Be (23^{\circ}C) = \text{Maximum Density Variation } ^{\circ}Be$$

$$u_l = (-0.259509373174239 ^{\circ}Be) - (-0.356431364170646 ^{\circ}Be) \\ = 0.0969219900996407 ^{\circ}Be$$

4. Scaling to a common statistical distribution

Since the uncertainty of the standard has already been divided by the k factor and the standard deviation of the process already has a sigma (σ) equal to 1, only the uncertainty due to liquid density variation must be scaled.

To scale this component half of the range is divided by the square root of three because it is assumed to have a rectangular distribution.

$$u_l = 0.0969219900996407 \text{ } ^\circ\text{Be} \times \frac{1}{2} \div \sqrt{3} = 0.027978968537211 \text{ } ^\circ\text{Be}$$

The final converted and scaled components are in the table below.

Description	Abbreviation	Quantity
Standard Deviation of the Process	s_p	0.023786443 $^\circ\text{Be}$
Uncertainty of the Standard	u_s	0.018 $^\circ\text{Be}$
Liquid Density Variation	u_l	0.027978968537211 $^\circ\text{Be}$

5. Combining the components

Once all the uncertainty components are in the same units, they can be combined using the root sum squared method.

$$u_c = \sqrt{s_p^2 + u_s^2 + u_l^2}$$

$$u_c = \sqrt{0.023786443^2 + 0.018^2 + 0.027978968537211^2} = 0.04089764725505 \text{ } ^\circ\text{Be}$$

6. Expanding and rounding

To expand the uncertainty the coverage factor must be determined using the degrees of freedom (df) from the measurement process, in this case the laboratory has 83 measurements documented on a control chart.

$$df = N - 1$$

$$df = 83 - 1 = 82$$

The coverage factor may be looked up in a table such as [NISTIR 6969](#) SOP 29 (2019) Appendix A or calculated using the T.INV.T2 function in excel for a 95.45 % confidence interval for 82 degrees of freedom. In this case if Appendix A is used the coverage factor would be 2.05 and using Excel yields a value of 2.03. It is acceptable to use either value.

$$U = u_c \times k$$

$$U = 0.04089764725505 \text{ } ^\circ\text{Be} \times 2.03 = 0.083022223927751 \text{ } ^\circ\text{Be}$$

Up to this point all calculations should have been completed without any rounding, but once the final expanded uncertainty has been calculated, the value is rounded to 2 significant figures.

$$U = 0.083022223927751 \text{ } ^\circ\text{Be} \rightarrow 0.08 \text{ } ^\circ\text{Be}$$

7. Evaluating

Once a final expanded uncertainty has been calculated, it needs to be evaluated to the procedure being used and the tolerance (maximum permissible error) allowed. For most mass measurements the expanded uncertainty

must be less than 1/3 of the tolerance of the artifact, and for volume measurement less than the tolerance specified in [NIST Handbook 105-3](#) Section 8.2.

8. Reporting

This calculated uncertainty, confidence interval, and a brief description of how it was estimated must be reported on all calibration certificates along with the calibrated value for each artifact.

How to evaluate the uncertainty to the procedure?

Most established procedures have specific criteria that the expanded uncertainty must meet for the use of that procedure to be valid, but importantly the expanded uncertainty should make sense based on the measurements that have been made to create it.

For example, at the Echelon 3 level in mass, the expanded uncertainty must be less than 1/3rd of the tolerance which the laboratory is applying to an artifact.

$$P_n = \frac{U}{\frac{1}{3} \times Tolerance}$$

This test is called a normalized precision test (P_n). For the expanded uncertainty to comply with this requirement, the absolute value of P_n must be less than 1.

$$P_n < 1$$

Tips from metrologists around the country

- Ensure all uncertainty components listed in the SOP are accounted for and any others you found to be significant
- Larger uncertainties don't mean worse measurements. Unrealistically small uncertainties are more problematic.
- Document where all components are derived from
- Don't confuse uncertainty with correction

Additional Resources

[International Vocabulary of Metrology 2012 edition \(VIM\)](#)

[Guide to the Expression of Uncertainty in Measurement \(GUM\)](#)

[NISTIR 6969](#) SOP 29 (2019)

[NISTIR 6969](#) GLP 9 (2019)

NIST Webinar: Basic Uncertainty Calculations

[NIST Handbook 105-3](#)