Determining filling volumes based on density measurement

Relevant for: Industries performing the conversion of density results into filling volumes, e.g. beverage, petroleum and chemical industries

Whereas some products are quantified by their weight, for example some pre-packed goods in Europe, others are filled, paid and even taxed by their volume. Frequently, it is easier to measure the weight of a fluid product with a balance, followed by calculating the filling volume by using the density of the sample. This procedure is well proven and straightforward and therefore frequently applied wherever the measurement of volumes is not possible.

1 Does "Density" always mean the same?

1.1 A few considerations

The density of liquids and gases is highly temperature-dependent. As a consequence, precise density measurements either have to be temperature-controlled or they require an accurate temperature measurement. The true density of liquids and gases in Anton Paar density meters is measured with the oscillating U-tube method.

1.2 The oscillating U-tube method

The sample is introduced into a U-shaped borosilicate glass tube that is being excited to vibrate at its characteristic frequency electronically. The characteristic frequency changes depending on the density of the sample. Through determination of the characteristic frequency, the density of the sample can be calculated.

1.3 True density

The true density ρ in kg/m³ or g/cm³ of a liquid is defined as its mass m divided by its volume V.

$$\rho = \frac{m}{V}$$

The mass m corresponds to the weight in vacuum and is independent of external conditions such as buoyancy in air or gravity.

1.4 Apparent density

The apparent density ρ_{app} of a sample is defined as the weight in air W divided by the sample's volume V.

$$\rho_{app} = \frac{W}{V}$$

The values of true and apparent density are different, even if their units are identical. The true density of air at 20 °C as measured in a density meter is 0.0012 g/cm³ whereas the apparent density of air at 20 °C is 0.0000 g/cm³ - air on a balance does not give a reading!

For the determination of filling volumes with a balance from a density result, the apparent density is the required value.

1.5 Converting true into apparent density

The apparent density ρ_{app} is defined as

$$\rho_{app} = \frac{\rho_{true,sample} - \rho_{air}}{1 - \frac{\rho_{air}}{\rho_{steel\ or\ brass}}}$$

where

ρ_{app} = apparent density of the sample;

ρ_{steel} = 8.0 g/cm³; ρ_{brass} = 8.4 g/cm³;

ρ_{air} = true density of air (≈ 0.0012 g/cm³);

$\rho_{true,\ sample}$ = true density of the sample.

The apparent density is smaller than the true density and can be calculated from the true density considering the buoyancy of the sample in air and the weight and density of a reference weight in steel or brass.

Today, steel is generally considered as the reference.

2 How to convert true density results?

2.1 Converting results obtained with DMA 35

2.1.1 Determining the temperature coefficient

The portable density meter DMA 35 (**Figure 1**) measures the true density at sample temperature and allows converting the result to the density at a reference temperature by means of a simple method modification which requires a temperature coefficient.

Figure 1: DMA 35 portable Density Meter

To determine the temperature coefficient, measure the true densities ρ_1 and ρ_2 at two different temperatures T_1 and T_2, and divide the density difference by the difference in temperatures.

$$Temperature\ coefficient = \left|\frac{\rho_1 - \rho_2}{T_1 - T_2}\right|$$

The result is the temperature coefficient. It always carries a positive sign.

2.1.2 Creating the method "apparent density"

The calculation of apparent density is carried out in two steps:

Step 1:
Select the measuring parameter "Density@" and enter the desired reference temperature and the appropriate temperature coefficient.

Step 2:
Enter an offset (density of air) to compensate for the buoyancy.

2.2 Converting results on the benchtop density meters

The software of DMA 501 (**Figure 2**) and DMA 1001 as well as the software of DMA 4101 / 4501 / 5001 (**Figure 3**) allows displaying the apparent density value directly by selecting the measuring quantities “App. Density Brass (or Steel)” in one of the output fields.

Figure 2: DMA 501 compact density meter

3 How to determine the filling volume?

The filling volume V of a certain sample weight can be calculated based on the sample’s apparent density.

$$V = \frac{W}{\rho_{app}}$$

The volume can be shown directly on a DMA display. To add a new custom function to a DMA 35 density meter, the required coefficients for the calculation need to be transferred to the instrument whereas DMA 501 / 1001 / 4101 / 4501 / 5001 density meters allow creating a custom quantity directly. The procedures of how to display the desired value are described in detail in the respective DMA manual.

Figure 3: DMA 4101 / 4501 / 5001

4 How to determine the accuracy of the calculated filling volume?

The accuracy of the calculated filling volume ΔV depends on the density of the solution and the accuracy of the instrument Δρ.

$$\Delta V = \frac{\Delta\rho}{\rho_{app,sample} + \Delta\rho} * 100\% \approx \frac{\Delta\rho}{\rho_{app,sample}} * 100\%$$

Table 1 lists the specified accuracies of various DMA density meters.

Table 1: Accuracies of various DMA density meters

Instrument	Specified instrument accuracy Δρ [g/cm³]
DMA 35	±0.001
DMA 501	±0.001
DMA 1001	±0.0001
DMA 4101	±0.0001
DMA 4501	±0.00005
DMA 5001	±0.000005

Example:

The accuracy of DMA 35 is 0.001 g/cm³ in the viscosity range < 300 mPa·s and the density range 0 g/cm³ to 2 g/cm³. Thus, the accuracy of the calculated filling volume ΔV for a density value of 0.7 g/cm³ is

$$\Delta V \approx \pm\frac{0.001\ g/cm^3}{0.7\ g/cm^3} * 100\% = \pm 0.143\%$$

If better accuracies are required, other density meters such as DMA 1001 / 4101 / 4501 / 5001 are recommended.

Table 2 summarizes the accuracies for samples with various densities when measured with different density meters. This table also advises which density meter is best suited for the required accuracy.

Table 2: Accuracies of calculated filling volumes

Density [g/cm³]	DMA 35; DMA 501 [%]	DMA 1001; DMA 4101 [%]	DMA 4501 [%]	DMA 5001 [%]
0.7	±0.143	±0.0143	±0.00714	±0.001000
1	±0.100	±0.0100	±0.00500	±0.000700
1.3	±0.077	±0.0077	±0.00385	±0.000538
1.6	±0.063	±0.0063	±0.00312	±0.000438
2	±0.050	±0.0050	±0.00250	±0.000350
2.3	DMA 501 only: ±0.043	±0.0043	±0.00217	±0.000304

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